Gravitational Lensing
Commentary and Discussion

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Provide commentaries to the following two papers:
Chris Kochanek: Turning AGN Microlensing from a Curiosity into a Tool
Gary Bernstein: Statistical Challenges of Weak Gravitational Lensing
Thanks to Chris and Gary’s papers on gravitational lensing, which vividly refreshed my memory of Pete Kernan’s “trick” on messing up someone’s picture:

Produced by lensing Gary’s picture on

What’s Gravitational Lensing (GL)

Massive bodies can bend/deflect the path of light rays

This effect is called GL

- **Strong Lensing**
  - magnifies, distorts w. obvious traces and may produce multiple images

- **Weak Lensing**
  - produces subtle distortion (shear, magnification)
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Strong Lensing

Weak Lensing

Gravitational Lensing – p.4/11
Challenges

Strong Lensing: Advertising → Delivering

There are data for probing DM, revealing M of distant galaxies, and resolving R, the internal structure of quasars.

Nd: not enough data and analyses based on the static assumptions.

Weak Lensing: regular/Gaussian → irregular/Non-Gaussian

How to measure distortion when shapes of galaxies are irregular

How to extract info given the intervening mass dist. is non Gaussian

Mine: Same AN: GR, AGN, ...,
Challenges

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**AN:** GR, AGN, ...,
**Strong Lensing by Chris**

Idea: \[ \text{data } D \rightarrow P(D|p) \text{ using } \chi^2 \text{ statistics} \]
+ prior on \( p \) \[ \rightarrow P(p|D) \]

where \( p = (k, k_*, \gamma, < M >, R_\lambda, \nu_e) \), \( k \) is mean surface density, \( k_* \) is surface density in stars, \( \gamma \) is shear.

1. Generate random magnification patterns on a range of \((k, k_*, \gamma, < M >)\)
2. Convolve w. selected disk models \( R_\lambda \)
3. Generate light curves \((\nu_e +...)\)
4. Compute the \( \chi^2 \) value for each light structure based on a threshold value to determine \( P(p) \)
   - Model selection vs. hypothesis testing (what are nulls)
   - Hierarchical models (ok) and final verification (data)
5. Approximate Bayesian integrals by sampling/MCMC
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- Laplace approximation
- Model averaging and biases

Random Thoughts and comments:
Extrapolation: Fig 2, data
Pages 6, 7, 8, 9
Comments A1-9
1. Define Shapes $e_i$ to galaxies that are irregular

Gary’s solu:

$$\hat{\gamma} = \frac{\langle e \rangle}{1 - \langle e^2 \rangle / 2}$$

where noiseless $e^2$ is unobserved.

- This leads to a measurement error models in statistics and need to do deconvolution. Is the error distribution (PSF) known?
- An alternative solution may be possible.
Suggestions:

- **Classify patterns** after a Procrustes analysis (affine transformation or image registration) of the data —> using shape code and Procrustes parameters

- **Use topology** (Euler characteristics)

- **Use a mixture** of known shapes with some wavelets basis or other orthogonal basis (Laguerre expansion)
  
  biases and distribution <— bootstrap
2. Characterize the mass-galaxy likelihood $L(m, g)$
Use a mixture of Gaussian distributions

3. Find a feasible analyses scheme
Use the adaptive estimates, PEM
by Sun, Liu and Chen (06)
Conclusion

For statisticians and scientists:

- **Know context**
  - *Who?* Individuals measured and observed
  - *What?* has been measured and observed
  - *Why?* Study Purpose

- **Have good designs**

- **Avoid bias**

- **Do something about the bias** if there is one.
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For scientists:

- Involve a statistician or use statistical strategies **early** (from the design of an experiment to the analysis of the resulting data) not just later (for the analysis part only).