



# ***Gravitational Lensing***

## ***Commentary and Discussion***

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Provide commentaries on  
the following two papers:  
Chris Kochanek: Turning  
AGN Microlensing from a  
Curiosity into a Tool  
Gary Bernstein: Statistical  
Challenges of Weak  
Gravitational Lensing

# Wonder of Gravitational Lensing



Thanks to Chris and Gary's papers on gravitational lensing, which vividly refreshed my memory of Pete Kernan's "trick" on messing up someone's picture:



♣ Produced by lensing Gary's picture on

<http://theory2.phys.cwru.edu/~pete/GravitationalLens/GravitationalLens.html>

# What's Gravitational Lensing (GL)



**Massive bodies can bend/deflect  
the path of light rays**

**This effect is called GL**

**magnifies, distorts w.  
obvious traces and  
may produce multiple  
images**

**Strong Lensing**

**produces subtle  
distortion  
(shear, magnification)**

**Weak Lensing**

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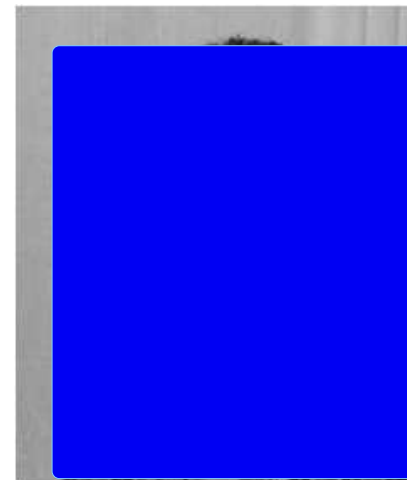
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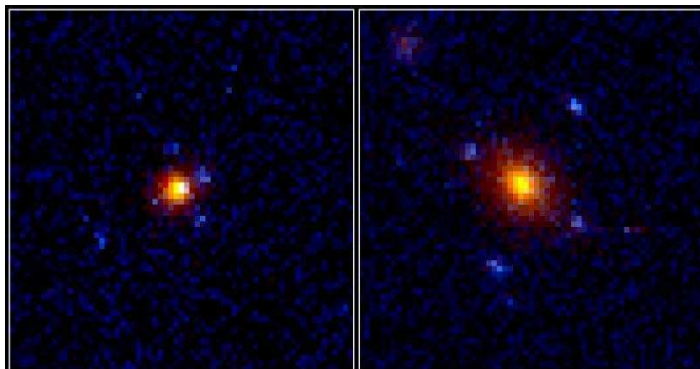
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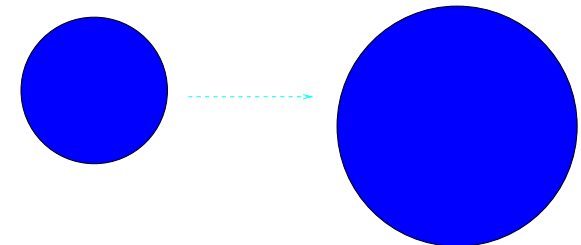
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Gravitational Lenses

HST · WFPC2

PRC95-43 · ST ScI OPO · October 18, 1995 · K. Ratnatunga (JHU), NASA





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**AN:** GR, AGN, ...,



Idea: data  $D$   $\rightarrow$   $P(D|p)$  using  $\chi^2$  statistics  
+ prior on  $p$   $\rightarrow$   $P(p|D)$

where  $p = (k, k_*, \gamma, \langle M \rangle, R_\lambda, \nu_e)$ ,  $k$  is mean surface density,  $k_*$  is surface density in stars,  $\gamma$  is shear.

1. Generate random magnification patterns on a range of  $(k, k_*, \gamma, \langle M \rangle)$
2. Convolve w. selected disk models  $R_\lambda$
3. Generate light curves ( $\nu_e + \dots$ )
4. Compute the  $\chi^2$  value for each light structure based on a **threshold** value to determine  $P(p)$ 
  - Model selection vs. hypothesis testing (what are nulls)
  - Hierarchical models (ok) and final verification (data)
5. Approximate Bayesian integrals by sampling/MCMC



## 5. Approximate Bayesian integrals by sampling/MCMC

- Laplace approximation
- Model averaging and biases

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### ● Random Thoughts and comments:

Extrapolation: Fig 2, data

Pages 6, 7, 8, 9

Comments A1-9



1. Define Shapes  $e_i$  to galaxies that are irregular  
Gary's solu:

$$\hat{\gamma} = \frac{\langle \mathbf{e} \rangle}{1 - \langle e^2 \rangle / 2}$$

where noiseless  $e^2$  is unobserved.

- This leads to a measurement error models in statistics and need to do deconvolution. Is the error distribution (PSF) known?
- An alternative solution may be possible.



## Suggestions:

- **Classify patterns** after a Procrustes analysis (affine transformation or image registration) of the data  
—> using shape code and Procrustes parameters
- **Use topology** (Euler characteristics)
- **Use a mixture** of know shapes with some wavelets basis or other orthogonal basis (Laguerre expansion)  
biases and distribution <— bootstrap





2. Characterize the mass-galaxy likelihood  $L(m, g)$

Use a mixture of Gaussian distributions

3. Find a feasible analyses scheme

Use the adaptive estimates, PEM  
by Sun, Liu and Chen (06)

# Conclusion



For statisticians and scientists:

- Know context
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  - Why? Study Purpose
- Have good designs
- Avoid bias
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For scientists:

- Involve a statistician or use statistical strategies early (from the design of an experiment to the analysis of the resulting data) not just later (for the analysis part only).